XX. Neopythagoreanism and Mathematical Symmetry in Lucan, *De bello civili* 1

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Quod cladis genus, o superi, qua peste paratis
saevitiam? extremi multorum tempus in unum
convenere dies. summo si frigida caelo
stella nocens nigros Saturni accenderet ignis,
Deucalioneos fudisset Aquarius imbres
totaque diffuso latuisset in aequore tellus.
si saevum radiis Nemeaeum, Phoebe, Leonem
nunc premeres, toto fluerent incendia mundo
succensusque tuis flagrasset curribus aether.
hi cessant ignes. tu, qui flagrante minacem
Scorpion incendis cauda chelasque peruris,
quid tantum, Gradive, paras? nam mitis in alto
Iuppiter occasu premitur, Venerisque salubre
sidus hebet, motuque celer Cyllenius haeret,
et caelum Mars solus habet. cur signa meatus
deseruere suos mundoque obscura feruntur,
ensiferi nimium fulget latus Orionis?
inminet armorum rabies, ferrique potestas
confundet ius omne manu, scelerique nefando
nomen erit virtus, multosque exibit in annos
hic furor. et superos quid prodest poscere finem?

Tension was high in Rome as Caesar advanced from Ariminum, and these despairing words of P. Nigidius Figulus, polymath, astrologer, and revivalist of Pythagoreanism,[1] were not intended by Lucan to lessen it. Ostensibly addressed to the gods, as the first and last lines of the quotation show, they were in effect an appeal to the *divinae artes*[2] of what Housman once called "that

[1] Cicero, *Timaeus* 1.1: "denique sic iudico, post illos nobiles Pythagoreos, quorum disciplina exstincta est quodam modo, . . . hunc exstitisse qui illam renovaret."

[2] Manilius 1.1.

intricate fraud by which Asia revenged herself on Europe for the conquests of Alexander."[3] The hard core of the passage, which is itself only a part of Figulus' speech, occurs in lines 651–63 with their astrological account of the sun and the five planets and the subsequent question in lines 663–65 about the unaccountably erratic behavior and dimness of the *signa* (presumably the signs of the zodiac) in evident contrast with the excessive brilliance of Orion.

It was noted by Boll, who made some attempt to comprehend these difficulties, that even Johann Kepler was frustrated in an effort which he once made in the interest of chronology to understand this passage.[4] Boll was concluding his chapter on the *Sphaera Graecanica* and the *Sphaera Barbarica* of Figulus and, in conformity with his texts of Teucer Babylonius earlier in his *Sphaera*, mentioned that Orion was one of the extra-zodiacal *paranatellonta*,[5] while he related its significance as *ensifer* or *xiphêrês* to the ensuing words of Figulus: "inminet armorum rabies . . . ius omne manu." As for the sun and the five planets, he tacitly followed the earlier and traditional explanation of lines 651–54 that Saturn was not in Aquarius; otherwise there would have been a *kataklysmos* like that of Deucalion.[6] Nor was the sun in Leo;

[3] A. E. Housman, "Astrology in Dracontius," *CQ* 4 (1910) 191.

[4] Franz Boll, *Sphaera* (Leipzig 1903) 362–63. It would seem from note 1 *ad loc.* that Boll's secondary source was F. von P. Schrank (ed.), *Sammlung naturhistorischer und physicalischer Aufsätze* (Nuremberg 1796) 233–301. For Kepler's correspondence on this subject with the Bavarian Chancellor Herwart von Hohenburg in 1597, 1599, and 1605–6 see vols. 13, 14, and 15 of his *Gesammelte Werke* in Max Caspar's edition (Munich 1945, 1949, and 1951), nos. 74, 78, 83, 121, 123, 352, 361, and 368. Kepler, who regarded Lucan's astrology as unrealistic, concluded in the last letter: ". . . frustra nos in Lucano explicando desudare, vt qui non ex caelo loquatur, sed ex Astrologorum regulis, et opinione sua" (vol. 15.295).

[5] Orion was relevant to both the *Sphaera Graecanica* and the *Sphaera Barbarica*. W. Gundel, *RE* 18.3 (1949) 1239, s.v. "Paranatellonta," refers to his own "Neue astrologische Texte des Hermes Trismegistos," *Abh. Akad. Münch.* 12, n.f., (1936) and to the regular Osiris-Orion equation of the Egyptians.

[6] See, among the later exponents of this explanation, the editions of Lucan by F. Oudendorp (Leyden 1728) and P.-A. Lemaire (Paris 1830). That it should be subsequently ignored by commentators, for example by C. E. Haskins (London 1887), is perhaps typical of a later age which knew little and cared less about the feasibility of such subtleties. However Boll, like his predecessors, saw in the sun's absence from Leo the corollary that Saturn was not in Aquarius, for this was a ready inference both from 653 and from the elementary fact that in the zodiac Aquarius is directly opposite Leo. As for 653, Aquarius was identified with Deucalion, e.g. by Hegesianax according to Hyginus, *Poet. astron.* 2.29, and by Germanicus, *Aratea* 562. (The *Commenta Bernensia*, ed. H. Usener [Leipzig 1869] 43, on 653 and 658 imply much of all this.)

otherwise he would have caused an *ekpyrôsis*. Mars of course was in Scorpio and Mercury was stationary, but Boll could do no more for Jupiter and Venus than to describe the one as "in der deiectio"[7] and the other as "machtlos." In passing, he mentioned Aquarius, Leo, and Scorpio as the astrological houses of Saturn, the sun, and Mars respectively, but he overlooked a further consideration which will be seen later to be of some importance, for he failed to specify that, among the houses, Aquarius was nocturnal and Leo and Scorpio diurnal.[8]

It remained for Housman to take an important step forward by consulting the Superintendent of the Nautical Almanac Office in London, on the assumption that Figulus' stellar observations were made "shortly after the arrival at Rome of the news that Caesar had crossed the Rubicon: say XVI kal. Febr. A.V.C. 705, answering to 28 Nov. 50 B.C. in our reformed calendar, when the sun was in 4° of Sagittarius." A calculation as to where the planets actually were in the heavens at that time revealed that Lucan's account was wrong about them all except Mercury in line 662, and Housman's assessment, in the Astronomical Appendix to his edition, of such truth and error as became apparent in this passage was highly informative so far as it went.[9]

In view, however, of the occasional precision of Lucan's technical terms, it was strange that Housman, who was editing him concurrently with Manilius, should have seen no more in this passage than an unsatisfactory and unrealistic account of planetary positions. More than twenty years ago it occurred to me that the language of Figulus was that of a *katarchê* or "investigation of the influence of a momentary configuration of the celestial

For Lucan's similar identification of Sagittarius with Chiron or, at any rate, one of the Centaurs, cf. 4.528: "nox tum Thessalicas urguebat parva sagittas."

[7] Jupiter could not have been in his *deiectio* or *tapeinôma*; cf. Housman's salutary remarks in the Astronomical Appendix to his edition of Lucan (Oxford 1926, 1950) 327.

[8] The planetary houses and their classification as diurnal or nocturnal were clearly set forth by A. Bouché-Leclercq, *L'Astrologie grecque* (Paris 1899) 182–92, and by Franz Boll and Carl Bezold, *Sternglaube und Sterndeutung*[4] ed. W. Gundel (Leipzig and Berlin 1931) 58–59. Apart from the sun and moon which had each one house (diurnal, of course, in the one case and nocturnal in the other), each of the five planets possessed two houses, one solar or diurnal and the other lunar or nocturnal. Lucan 1.651–63 is damaging to Wachsmuth's view that the planetary houses were not known to Figulus (see Bouché-Leclercq 185, note 2).

[9] *Op. cit.* (above, note 7) 325–27 for this and the quotation in my previous sentence.

objects, e.g. at the beginning of a journey, upon the outcome of an enterprise,"[10] or, specifically in the present case, at the commencement of the Civil War. In *katarchai*, as in horoscopes (which they would be called if, instead, they forecast human lives at birth), the twelve signs of the zodiac were thought of as rotating clockwise in diurnal motion through an imaginary circular band or wheel with a fixed framework. This circular band was called the *dôdecatropos* and was divided into twelve equal compartments which the Greeks called *topoi* and the Romans *templa*, *loca*, *sedes*, or *partes*. These were numbered from 1 to 12 in a counterclockwise direction, beginning from the *horoscopus* on the eastern horizon in the first *templum*. The *horoscopus* was the first of the four *cardines* or points which the Greeks called *kentra*, while the others, in the numerical order of the *templa*, were named *imum caelum* or IMC in the fourth *templum* below the earth, *occasus* in the seventh *templum* on the western horizon, and *medium caelum* or MC in the tenth *templum* at the culminating point of the zodiac in the visible heavens and diametrically opposite the IMC.[11]

So in my reconstruction, which was accompanied by a diagram, of Figulus' *katarchê*,[12] I placed Scorpio in the Ascendant or first *templum*, Sagittarius consequently in the second, Capricorn in the third, and so on. This stabilization of the moving zodiac within the fixed *dôdecatropos* at the moment of Figulus' observations is confirmed by the continued visibility of Orion (line 665), for the picture of the rising Scorpion putting to flight the setting giant who had tried to molest Artemis was a commonplace among catasterisms.[13] The insistence of C. S. Phloratos that Scorpio was the heavenly counterpart of Caesar and Orion of Pompey is a belief which I do not necessarily share, but it is less bizarre than his identifications of Jupiter, Venus, and Mercury with, respectively,

[10] O. Neugebauer and H. B. van Hoesen, *Greek Horoscopes* (Philadelphia 1959) 7. I had not then perused Kepler's blend of sagacious and abortive speculations (above, note 4) along similar lines. Housman too might have profited from them.

[11] The most convenient and lucid account of the *cardines* and the *dôdecatropos* is to be found in Housman's edition of Manilius' second book (London 1912; second edition with addenda, Cambridge 1937) xxvi–xxxi. Discussing the terms MC and IMC he remarked: "These, in the plain sense of the words, should mean the zenith and the nadir, but they actually signify only the highest and lowest points of the zodiac" (xxvii), i.e. as the zodiac appears to move, with the apparent diurnal motion of the sky, from east to west, from the *horoscopus* to the *occasus*.

[12] "The Astrology of P. Nigidius Figulus (Lucan I, 649–65)," *CQ* 35 (1941) 17–22.

[13] Aratus, *Phaenomena* 634–46.

Crassus, Julia, and Cicero.[14] Despite his conscious detachment from the idea of a *katarchê* such as I postulated, he virtually assumes it so far as Scorpio and Orion are concerned. Granted the time of the year, the fact that Orion had not yet wholly set, and what was said about Mars in lines 658–60 and 663, no other alignment of the zodiac within the *dôdecatropos* is possible.

With Mars in that part of Scorpio which had risen and, because of line 663, the only planet above the horizon, I argued that Mercury and Venus were imagined by Figulus as being together in Sagittarius and in the second *templum*, while Jupiter was in Aries and in the sixth *templum*.[15] But with regard to the remaining planet, my previous article requires a correction which is instructive. In Figulus' account the sun was not in Leo: therefore, according to the *katarchê*, it was not at MC. But Saturn had just been mentioned as not being *summo caelo*, which I had identified as MC.[16] This identification, together with my assumption for the sake of argument that the imperfect subjunctive *accenderet* (line 652) was intended to be *diametrically* contrary to fact, led me to place Saturn at IMC and in Aquarius. But there he would have brought about a disaster complementary to that of the sun in Leo,[17] and it was fortunate for the world that they were co-operators *in absentia* from the locations where they would have caused, respectively, a *kataklysmos* and an *ekpyrôsis* (lines 651–57). Their co-operation, inactive though it was (line 658: "hi cessant ignes"), points to a connection with the belief that Saturn was the planet of the sun, as we learn from *Epinomis* 987c: Ἡλίου δ' αὐτόν τινες ἐπωνυμίαν φθέγγονται.[18] Thus he was also the sun

[14] Χαράλ. Σ. Φλωράτου, Ἡ Προφητεία τοῦ P. *Nigidius Figulus* (*M. Annaei Lucani Belli Civilis* I. 639–673) (Athens 1958) 34–44, 50. On 25 he expresses his dissatisfaction with Housman and with me for regarding lines 663–65 as descriptive of an atmospheric rather than an astrological phenomenon. But see Cicero, *Div.* 1.57.130.

[15] The trigon of Aries, Leo, and Sagittarius would appear to have been relevant to the threat which Caesar offered, coming as he did from Transalpine and Cisalpine Gaul. Cf. Claudius Ptolemaeus, *Tetrabiblos* 2.3.59–60, where it was equated with the first of the four quarters of the *oikoumenê*, i.e. τὸ κατὰ τὴν Κελτογαλατίαν.

[16] Like the *Commenta Bernensia* (above, note 6) 43 and with some reason because of Manilius 2.810–11: "primus erit ⟨cardo⟩, summi qui regnat culmine caeli/et medium tenui partitur limite mundum," which I quoted, *op. cit.* (above, note 12) 19. Of MC and IMC, Housman (above, note 11) xxvii remarked: "To hear Manilius talk, 'tertius excelsi signat fastigia caeli,' 'ima tenet quartus fundato nobilis orbe,' you would fancy that the one were sheer overhead and the other plumb underfoot."

[17] See above, 311, and note 6.

[18] See also Diodorus 2.30.3 and other references cited by F. Cumont, *AntCl* 4

of the night,[19] and Figulus in his *katarchê* conveniently found his nocturnal house of Aquarius at IMC just as Leo, the house of the sun, was diametrically opposite at MC.[20] Could Aquarius and IMC in the middle of the infernal semicircle of the *dôdecatropos* be imagined as located *summo caelo*, even as Leo and MC might be so described in the center of the supernal half?

To answer this question, the first step is to cite such an authority as Sextus Empiricus who, in speaking of the four cardinal points or *kentra*, added to his mention of the *antimesouranêma* that it was also the *mesouranêma*.[21] The next is to turn to the well-known

(1935) 14, note 2, also Boll-Bezold (above, note 8) 5 and 48 for the Chaldaean origin of this belief. J. Bidez, *Éos ou Platon et l'orient* (Brussels 1945) 94, argued strongly in this and other connections that Philip of Opus and not Plato was the author of the *Epinomis*. He went on to say (183, note 7), as he had done earlier in *RevPhil* 29 (1905) 219, that the authority of the better MSS., A and the first hand of O, should not be set aside in order to replace *Hêliou* with *Kronou*. The false reading was substituted for the true, according to Bidez, in the Byzantine period when "on ne parvint plus à comprendre cette façon de présenter Saturne pour une ombre d'Hélios dans le ciel des enfers et des trépassés."

[19] Stefan Weinstock, "Martianus Capella and the Cosmic System of the Etruscans," *JRS* 36 (1946) 101–29, esp. 120, describes this as "of some relevance to the Etruscan doctrine" of the sixteen regions of the sky, for which see also Cicero, *Div.* 2.18.42 and Pease *ad loc.*, as well as Boll-Bezold (above, note 8) 155. In note 119 Weinstock mentions two articles by Cumont on Saturn in this connection, but see also E. des Places, "Platon et l'astronomie chaldéenne," *Mélanges Cumont* (Brussels 1936) 129–42, esp. 138–39, and the reprinted translation of Cumont's book under the title *Astrology and Religion among the Greeks and Romans* (New York 1960) 28. Weinstock 115 and note 84 agrees with a number of earlier scholars that Figulus rather than Varro was the ultimate source of Martianus Capella's discussion of the sixteen regions which, as Bouché-Leclercq, *Histoire de la divination dans l'antiquité* 4 (Paris 1882) 24 ff., showed, were comparable with the twelve *sortes* or *athla* of Manilius 3.43–202 rather than with the *templa*. For the *sortes* see Housman's edition of this book of Manilius (London 1916; second edition with addenda, Cambridge 1937) v–xi and Weinstock 103 and 108. Ragna Enking, "P. Vergilius Maro Vates Etruscus," *Mitteil. d. deutschen arch. Inst., Röm. Abt.*, 66 (1959) 65–96, esp. 84–85, finds an allusion to the sixteen regions of the Etruscan sky in Vergil's *domus* of the winds in *Georgics* 1.370–71.

[20] If Saturn had been in Aquarius and therefore at IMC, he would also have been in the fourth *templum*, which was his abode; see Housman (above, note 11) xxx and cf. the *Commenta Bernensia* (above, note 6) on 653: "Aquarius enim in notio orbe situs est qui est propria sedes Saturni."

[21] *Adv. math.* 5.12: *ἀντιμεσουράνημα, ὃ καὶ αὐτὸ μεσουράνημά ἐστιν.* The whole passage was quoted by Housman (above, note 11) xxvii–xxviii, just after he had remarked in *CQ* 5 (1911) 251: "I do not need to be told that *mesouranêma* is sometimes used (and correctly enough) for *antimesouranêma*." Cf. Claudius Ptolemaeus, *Tetrabiblos* 3.10.135: *τῆς μεσουρανούσης οἰκείως ἤτοι ὑπὲρ γῆν ἢ ὑπὸ γῆν μοίρας.* Thus in Manilius 2.892, where the fifth *templum* immediately to the west of MC is described and the MSS. display *imaque submersi contingent fulmina mundi*, Scaliger admirably conjectured *culmina* (preceded by *contingens*) for *fulmina*.

passage in Aristotle's *De caelo* where, said its author:

> It is clear, then, that the unseen pole is the upper, and those who live thereabouts are in the upper hemisphere and on the right, while we are in the lower hemisphere and on the left. The Pythagoreans say the opposite, for, while they place us above and in the part on the right, they put the others below and in that on the left. But the converse is the case.[22]

What was cosmically up and down, to the right and to the left, in this passage and elsewhere has been the subject of much discussion from Proclus to modern times.[23] But whatever the explanations, Aristotle at any rate described as the opposite of his own the Pythagorean view that we are in the upper hemisphere and on the right, while the antipodeans are in the lower hemisphere and to the left. This statement of his should be of interest to speculators on Pythagoreanism in Vergil, for the description of the zones in *Georgics* 1.231–51 differs from its original in the *Hermes* of Eratosthenes in that, while the latter referred to antipodeans and Vergil spoke of the north pole as high above us and of the south pole as beneath our feet, to the poet of the *Georgics* the frigid zones were (respectively, as it would appear) on the right and on the left. Of course Aratus had mentioned one pole as invisible and the other as facing it in the north and high above the ocean, but, like Eratosthenes (so far as we can tell from a fragment), he added nothing about right and left.[24] Inasmuch as the five celestial zones were known to Pythagoras and his followers,[25] the question

[22] 2.2.285B, 22–27.

[23] See the Loeb edition by W. K. C. Guthrie, *Aristotle on the Heavens* (1945) 136–39, Weinstock (above, note 19) 119–20 and note 111, and Benedict Einarson's review article, *CP* 53 (1958) 91–99, esp. 91–92, on the Budé edition of the *Epinomis* by E. des Places, S.J. (Paris 1956). Weinstock 120 and note 117 mentions as probably due to the Pythagoreans Plato's doctrine: "when the souls are judged, the just are sent to the right and up through the heavens, the unjust to the left and down."

[24] *Phaenomena* 24–26. For the *Hermes* see Friedrich Solmsen, "Eratosthenes as Platonist and Poet," *TAPA* 73 (1942) 192–213, esp. 200: "Eratosthenes is not known to have taken an interest in Pythagorean motifs" and 207: "Pythagoreanism, in all likelihood, affected Eratosthenes only to the extent to which it had been assimilated by the Platonic school."

[25] W. A. Heidel, "The Pythagoreans and Greek Mathematics," *AJP* 61 (1940) 1–33, observed (14, note 24): "Aëtius, II, 12, 1 says that Pythagoras and his followers knew the five celestial zones. This looks like another inference to Pythagoras from (late) Pythagoreans, such as we might expect Posidonius to make." For Aëtius see Diels-Kranz, *Vorsokr.* 1.77.30–35 (*Dox. Graec.* 340) and, for the correspondence of the celestial and terrestrial zones in Pythagorean theory, Hugo Berger, *Geschichte der wissenschaftlichen Erdkunde der Griechen*[2] (Leipzig 1903) 206–7.

should now be asked whether in astrology descriptions of the *antimesouranêma* or IMC, which some astrologers thought of as northern,[26] as if it were the *mesouranêma* or MC or, as Figulus was described by Lucan as having called it, the *summum caelum*, were not either of Pythagorean origin or of Pythagorean belief.[27]

As for other traces of Pythagoreanism or Neopythagoreanism in Lucan's complete passage about Figulus, there is the *numeris moventibus astra* of line 641, where the *sequentibus* of Bentley, followed by Housman,[28] is invalidated, for example, by Aristotle, who described the so-called Pythagoreans as supposing the whole heaven to be harmony and number.[29] Phloratos has also commented on the belief in one cosmos which was held by Pythagoras and Ecphantus as well as by others, and also on the destruction of this cosmos by conflagration or by cataclysm.[30]

To turn from the content of the whole episode of Figulus to its form, it is contained in 34 lines (639–72), while his despairing appeal to the gods occupies the 21 lines (649–69) which are prefaced to this article. George E. Duckworth has pointed out in the present volume that 21 and 34 are the first numbers in the Fibonacci series to produce the ratio .618,[31] and with Lucan's passage

[26] See Bouché-Leclercq (above, note 8) 272 and note 2 as well as Housman (above, note 11) xxvii.

[27] Cf. Boll-Bezold (above, note 8) 155–56.

[28] Franz Blatt, "Lucan and his Text," *Classica et mediaevalia* 20 (1959) 47–67, is in favor of greater caution than was shown by Bentley, Housman, and others in emending a number of passages in the *Bellum civile* as a whole and in the first book in particular. Bentley's wish to alter *Saturni* to *Capricorni* in line 652 (possibly because Capricorn was Saturn's solar or diurnal house) would have meant an emendation as violent and erroneous as *sequentibus* for *moventibus*.

[29] *Metaphysics* A 5.986A, 1–3 and 15–21: see also, for example, Sextus Empiricus, *Adv. math.* 4.2, and the *Commenta Bernensia* (above, note 6) 42 on Lucan 1.640: ". . . sive secundum Pythagoram, qui omnia ratione conposita et numeris constare confirmat." Werner Jaeger, *Aristotle*[2] translated by Richard Robinson (Oxford 1948) 455, remarks: "When Aristotle speaks repeatedly of the 'so-called Pythagoreans,' he means the scientific circle of Archytas of Tarentum, with whom Plato had personal intercourse."

[30] *Op. cit.* (above, note 14) 15, citing Diels-Kranz, *Vorsokr.* 1.442.19–20 (*Dox. Graec.* 327) and 1.404.1–3 (ibid. 333) for Aëtius 2.1.2 and 2.5.3 respectively. For Figulus' cautious use in his *De diis* of Posidonius in connection with the *ekpyrôsis* see J. Geffcken, "Die Hirten auf dem Felde," *Hermes* 49 (1914) 321–51, esp. 327 and 337. As for the *De diis*, Theodor Birt, "*Ἄγνωστοι θεοί* und die Areopagrede des Apostels Paulus," *RhM* 69 (1914) 342–92, esp. 386–87, observed that it, like the *Sphaera Graecanica* and *Sphaera Barbarica*, is referred to by Lucan, 1.639–40, in his introduction of Figulus: "cui cura deos secretaque caeli nosse fuit."

[31] "Mathematical Symmetry in Vergil's *Aeneid*," *TAPA* 91 (1960) 193 f. For the likelihood that the Fibonacci series was Pythagorean and known to Eudoxus see, for

about Figulus the Vergilian arrangement of *Georgics* 1.204–58 should be compared.[32]

Of the 55 lines of the Arruns episode (584–638), which immediately precedes that of Figulus, the first 21 (584–604) describe that soothsayer's instructions to others, while the last 34 (605–38) tell what Arruns himself did and said. The Arruns and Figulus passages are closely linked, inasmuch as there was an undoubted relationship between Figulus' Neopythagoreanism and the *Etrusca disciplina* of such authorities as his contemporary and, presumably, his fellow-exile, Aulus Caecina.[33]

So with the figure 55 the Fibonacci series is continued, and one may begin to wonder whether Lucan's first book was a tour de force similar to that which Le Grelle has seen in the first book of the *Georgics* and Duckworth now sees in the *Aeneid*. The arrangement of the book as a whole is remarkable. Editors print it with a numeration of 695 lines, but 436–40 comprise interpolations of much later date[34]; therefore the total is 690. The book divides where the catalogue of the provenience of Caesar's soldiers ends, after line 465, with the first part describing the outbreak of the Civil War in addition to its causes, while the second deals with

example, W. W. Rouse Ball, *A Short Account of the History of Mathematics*[4] (London 1908, 1940) 44–45; B. L. van der Waerden, *Science Awakening*, Eng. trans. by Arnold Dresden (Groningen 1954) 90 and 184; Vincenzo Capparelli, "Ludus Pythagoricus e divina proporzione," *Sophia* 26 (1958) 197–210, esp. 206–8 entitled "La divina proporzione nell'antichità"; and J. F. Scott, *A History of Mathematics from Antiquity to the Beginning of the Nineteenth Century* (London 1958) 23.

[32] Where (cf. above, 316) the zones are described in 21 lines (231–51). The 55 lines are the "foyer astronomique" of Guy Le Grelle, S.J., "Le premier livre des Géorgiques, poème pythagoricien," *Les études class.* 17 (1949) 139–235. When Lucan was composing his address to Nero in 34 lines (33–66), he had in mind Vergil's invocation of Octavian in the first book of the *Georgics*. See Sven Eriksson, *Wochentagsgötter, Mond und Tierkreis* (Stud. Gr. et Lat. Gothoburgensia 3; Stockholm 1956) 88–95, esp. 91. Eriksson is critical of J. D. Duff and of me for having misunderstood *orbe medio* in line 58 as "at the centre of the system" and "in the zodiac" respectively, but he has overlooked *Phoenix* 5 (1951) 106 (=*Studies in Honour of Gilbert Norwood* [Toronto 1952] 182) note 12, where I have expressed myself more clearly.

[33] See Leonardo Ferrero, *Storia del Pitagorismo nel mondo romano* (Turin 1955) 295–97, and for Aulus Caecina (father and son) Henry Bardon, *La littérature latine inconnue* 1 (Paris 1952) 314–16. Figulus' knowledge of Tages, the *conditor artis* of lines 636–37, was attested by Ioannes Laurentius Lydus, *De ostentis* 27–38 (Wachsmuth); cf. M. Schanz—C. Hosius, *Geschichte der römischen Literatur* 1[4] (Munich 1927) 554, and W. Kroll, *RE* 17.1 (1936) 207–8, s.v. "Nigidius."

[34] See, for example, the note *ad loc.* of Housman (above, note 7) 19, who omitted all five lines from his text, and also Ch. W. Whitaker, "Lucan and the Loire," *Mnemosyne* 9, ser. 4, (1956) 320–24.

the effect in Rome.[35] Thus the first part is composed of 460 and the second of 230 lines in an exact 2 + 1 = 3 ratio. Now the flattering address to Nero in lines 33–66 is not concerned with the beginning of the Civil War, but with its ultimate consequences; cf. lines 44–45: "multum Roma tamen debet civilibus armis / quod tibi res acta est." So, if the 34 lines of this piece of flattery are added to the 230 lines of the second part of the book (466–695), the addition may be justified by Figulus' remark about the principate, if not the tyranny of Julius Caesar, in line 670: "cum domino pax ista venit."[36] The result is a total of 264 lines for the second and minor portion of the book as against the 426 of the first and major. The ratio of the major (426) to the total (690) is .617, and that of the minor (264) to the major (426) is .620. Also 264 is to 426 as 132 is to 213, the total of which is, of course, half of 690 or 345. What a Pythagorean look this has![37]

Nor is this all. There are other Golden Mean ratios in Lucan's first book, e.g. one section ends at line 97 and another at line 157, and within the 60 lines of this passage the true division would seem to come between the 23 lines of 98–120 and the 37 lines of 121–57.[38] More might be adduced, not in an attempt to convince the sceptical, but in order to emphasize that Lucan in this

[35] The first part is divisible after 182, so that the whole book, like those of the *Aeneid* (see Duckworth [above, note 31] 190) really has three sections, the first being devoted to the causes of the Civil War (1–182) and the second to its prelude as Caesar crossed the Rubicon and occupied Ariminum. Otto Schönberger, "Zur Komposition des Lucan," *Hermes* 85 (1957) 251–54, esp. 251, assumed the following divisions: 1–223, 224–465, 466–695, of, respectively, 223, 241 and 230 lines. How Schönberger arrived at his figures for his first and second sections I do not know, but he is orthodox in considering that the third begins at 466 and is about events at Rome. Vittorio d'Agostino, "Personaggi ed eventi nel primo libro della Farsaglia di Lucano," *RivStudClass* 7 (1959) 266–76, divides the book, with reference to its ideas, into three parts (1–182, 183–521, 522–695) but, with reference to its action, makes a division after 468.

[36] Richard T. Bruère, "The Scope of Lucan's Historical Epic," *CP* 45 (1950) 217–35, esp. 227, observes, with respect to an earlier view of mine on this phrase: "...but Figulus is here thinking not of Julius Caesar but of Octavian."

[37] Vitruvius 9. praef. 6–7. For the so-called "nuptial figure" or "theorem of the bride" in Plato and elsewhere see, for example, James Adam, *The Republic of Plato* (Cambridge 1902) 264–312; Sir Thomas L. Heath, *The Thirteen Books of Euclid's Elements*[2] (London 1926, New York 1956) 1.417; and Arthur Ahlvers, *Zahl und Klang bei Platon* (Bern 1952) 11–20.

[38] Duckworth (above, note 31) 194. For the view that the "Golden Section" was the mediaeval expression for "the section" to which Proclus referred in his commentary on Lucan's first book, see George Sarton, *Isis* 42 (1951) 47, and van der Waerden (above, note 31) 184.

book seems to have honored his own introduction of the anti-Caesarian Figulus by paying some attention to a mathematical symmetry which he may have derived from Figulus himself, from Posidonius, or, for all we know, from Ennius.[39]

If sceptics wish to describe as *somnia Pythagorea* the conclusions which can be drawn from obvious and traditional paragraph divisions, they will find it difficult to ascribe so many arithmetical facts to coincidence.[40] Whatever Lucan did elsewhere, the internal evidence of his first book is strong. But, one may ask, is there any external evidence comparable with the statement in the *Vita Donati* that Vergil "maxime mathematicae operam dedit"?[41]

In chapter 58 of Petronius' *Satyricon* Hermeros apparently turns, after an obvious lacuna in the text, from his vituperation of Giton to Ascyltos with a remark which is thus given uniquely in the Codex Traguriensis: "non didici geometrias cretica et alogias menias, sed lipidarias litteras scio, partes centum dico ad aes, ad pondus, ad nummum." *Cretica* is as easily and as regularly emended to *critica* as *lipidarias* to *lapidarias*; but in this typical attempt of a man of little education to justify himself, what connection is there between *geometrias*, *critica*, and *alogias menias*, and what sense can be extracted from the corruption in the last of these three terms?

Friedlaender, who printed *alogias menias*, translated the first part of the sentence thus: "Ich habe keine Mathematiken und

[39] Horace seems to have paid a similar compliment to the memory of Archytas (cf. above, note 29) in *Odes* 1.28, where the sharp division after line 22 provides a major of 11 and a minor of 7 couplets with .611 as the ratio of the major to the total. (Duckworth [above, note 31] 211 has shown how striking is the inference from the *Ars poetica* that Horace's *aurea mediocritas* may have embraced Golden Mean ratios.) As for Posidonius, Frank Egleston Robbins, "Posidonius and the Sources of Pythagorean Arithmology," *CP* 15 (1920) 309–22, critically examined the belief that he was "the chief ancient authority upon the Pythagorean lore of numbers" and concluded: ". . . all that can be held proven is that Posidonius was interested in Pythagorean arithmology, and that he may have influenced some lines of its tradition." Posidonius' name cannot be omitted from a tentative list of sponsors of mathematical symmetry, although Arthur Darby Nock's article in *JRS* 49 (1959) 1–15 is salutary in demonstrating how tentative our approach to Posidonius must still be. For Posidonius, Neopythagoreanism, number-mysticism, and Figulus see Cumont (above, note 19) 49–50.

[40] From Burman (Leyden 1740) to Duff (Loeb Classical Library 1928) editors with few exceptions have ended paragraphs with lines 66, 97, 157, 465, 583, 638 and 672, as well as elsewhere.

[41] Cf. Duckworth (above, note 31) 194.

Ästhetiken und all den andern Unsinn gelernt."[42] Max Niedermann picked up a suggestion by F. Marx and rejected emendations such as *meras*, *naenias* and *Menenias* in order to justify *menias*, which he took to be the accusative plural of the first word of the *Iliad* and accordingly assumed for it the meaning of "Iliads" or "poems like the *Iliad*."[43] Thus, altering *alogias* to *alogas*, he produced the following version: "'Solche Lappalien,' sagt Hermeros, 'wie Mathematik und Philologie und alberne Gedichte wie die Ilias habe ich freilich nicht gelernt.'"

Some years earlier Evan T. Sage had commented as follows on the *alogias nenias* which he printed: "'senseless foolishness:' it is impossible to tell which is adj. and which is noun, though *alogias* is used as a noun in Sen. *Apoc.* 7:1."[44] And so it is here, although in a context where the word is mentioned along with *geometrias*, it must surely have the technical meaning of "irrationals" or "incommensurables," i.e. such arithmetizations of geometrical theorems as have been indicated in the linear symmetries of Lucan.[45] The word which may have been readily corrupted in the Traguriensis is not the noun *alogias* but the adjective *menias*, in which is slightly disguised a more obvious epithet than *Menenias* with its allusion to the obscure person mentioned by Horace.[46] Niedermann was on the right lines with the wrong idea, for *menias* is probably a corruption of *M(a)e(o)nias* in the sense of "epic."

[42] Ludwig Friedlaender, *Petronii Cena Trimalchionis*[2] (Leipzig 1906, Amsterdam 1960) 154–55.

[43] *PhilWoch* 56 (1936) 1294–95. These emendations were due respectively to J. Mentel (who called himself anagrammatically J. C. Tilebomenus), Heinsius (*nenias*), and Heraeus, while Marx's suggestion is to be found in the prolegomena to his *editio maior* of the *Auctor ad C. Herennium* (Leipzig 1894) 75.

[44] *Petronius: The Satiricon* (New York and London 1929) 171.

[45] An obvious inference from an attested meaning of the adjective *alogos*. The Fibonacci and similar series are such arithmetizations of geometrical theorems, i.e. Euclid 2.11, 6.30, and 13.1–6, for which see van der Waerden (above, note 31) 184. A. E. Taylor, "Plato and the Authorship of the *Epinomis*," *ProcBritAc* 15 (1929) 235–317, esp. 304, aptly remarked apropos *Epinomis* 990c5–e1: "The point is that, contrary to the current opinion, 'irrationals' *are* numbers, and that it is really a *numerical* problem we are investigating when we deal with a problem of plane or solid geometry. . . . The names γεωμετρία, στερεομετρία, obscure this important point, that the problems are really numerical." For irrationals, particularly $\sqrt{5}$ (1.618+ .618) see Plato, *Theaetetus* 147d and, most recently, Gustav Junge, "Von Hippasus bis Philolaus: Das Irrationale und die geometrischen Grundbegriffe," *Classica et mediaevalia* 19 (1958) 41–72, esp. 45–50, and A. Wasserstein, "Theaetetus and the History of the Theory of Numbers," *CQ* 8, n.s., (1958) 165–79.

[46] *Satires* 2.3.287. For Porphyrio *ad loc.* see Friedlaender (above, note 42) 302.

The innuendo of Hermeros' "geometries, aesthetics, and epic irrationalities" (no doubt with a typically Petronian *double entendre* in the last word) was directed, no doubt, against a recognizable feature of Lucan's first book.[47] If Horace could use this adjective in describing Varius, so could Petronius of Lucan.[48]

There are those who will be repelled by any suggestion that a poet like Vergil or even Lucan could have tempered his inspiration with mathematical symmetry, but they might consider the possibility that Harmonia was the mother of the Muses.[49] To Gaston Milhaud, writing in 1896, Euclid was an exponent of the Greek genius comparable to Homer, Plato, or Phidias[50]; and Vergil, in giving particular attention to *mathematica*, was aware of his full debt to Greece. As time goes on, the implications of this statement in the *Vita Donati* will be admitted more widely. If not, the critics of Duckworth's ingenious and indeed brilliant discoveries will have proved him to have been in error, not in the details which will inevitably be open to question, discussion, and possible improvement, but in principle. This is now their task.

My earlier article concluded with the opinion that Lucan 1.639–72 is a record of a prophecy by Nigidius Figulus which was actually delivered, but that several astronomical details were misrepresented by this eminent Neopythagorean in the interests of astrological charlatanry.[51] Together with some external considerations, the information vouchsafed by Figulus is, however, sufficient to permit a reconstruction of most of his *katarché*, where

[47] To Petronius this book was highly topical, as is shown by his obvious parody in *Sat.* 119–24. Jean Gricourt, "L'Ésus de Pétrone," *Latomus* 17 (1958) 102–9, concludes that the Hesus of *Sat.* 104 parodied the Esus of *Bell. civ.* 1.445.

[48] *Odes* 1.6.2: "Maeonii carminis alite." Whether Varius' *De morte* was concerned with Caesar or not, Henry Bardon, *La littérature latine inconnue* 2 (Paris 1956) 30, remarks of its fragments: ". . . l'énergie dense de la forme annonce Lucain, et la splendeur colorée, Virgile." As for *Maeonias*, Hermeros' Greek was such that he might have said *Menias*; cf. his *topanta* in his description of Trimalchio's wife, Fortunata, in chapter 37: "in caelum abiit et Trimalchionis topanta est."

[49] Or perhaps the daughter, for Euripides, *Medea* 830–32, is not wholly clear on this point. See Maximilian Mayer, *RE* 16.1 (1933) 695–96, s.v. "Musai," for this passage and a mention of Pythagoreanism. Anyhow Cicero made one of his characters in the *De republica* say that the Platonic Socrates aimed "numeros . . . et geometriam et harmoniam . . . Pythagorae more coniungere" (1.10.16). For Polyhymnia as the Muse of geometry and Euclid as her representative see Mayer 726.

[50] "La géométrie grecque considérée comme œuvre personnelle du génie grec," *REG* 9 (1896) 371–413. In this connection the wise words of Yves Béquignon, *Bull. de l'Assoc. G. Budé* 11, n.s. (Oct. 1950) 81–84, should be noted.

[51] See above, note 12.

summo caelo in line 651 is now seen to be a cosmological description, not of *medium caelum* in astrology, as I had previously thought, but of its counterpart *imum caelum*. Like other details in his speech, this unusual expression may represent a Neopythagorean point of view in a context so mathematically symmetrical that support is lent to the arguments of George E. Duckworth elsewhere in this volume concerning Vergil.[52] With a simple correction of a passing remark in *Satyricon* 58 by one of Petronius' characters, a covert allusion to Lucan's mathematical symmetry is detected. In such a special context as his first book, the internal evidence for this symmetry would seem again to be due to a Pythagorean tradition. Indeed, from Ennius onwards, the influence of Pythagoreanism or Neopythagoreanism upon Roman poetry may be imponderable, but it is far from being negligible.[53]

[52] See above, note 31. In a discussion of Le Grelle's article (above, note 32), Viktor Pöschl, *Anzeiger f.d. Altertumswiss.* 6 (1953) 1–4, remarked of the Golden Section between the "Works" (43–203) and the "Days" (204–463.5) in the *Georgics*: "Diese interessante Tatsache ist meiner Überzeugung nach ein Ausdruck für das in höchstem Masse ausgeprägte Gefühl des Dichters für Rhythmus und Proportion." But, while agreeing that Le Grelle's viewpoint is thus legitimate, he felt that it has nothing to do with Pythagoreanism. As far as Lucan's first book is concerned, my aim has been to draw the opposite conclusion. It may indeed be added that, in the arithmetic of its construction, the skillful union of the so-called "theorem of Pythagoras" in its simplest rational application (above, 319 and note 37) with what is, in effect, the irrational of division in extreme and mean ratio (above, 321 and note 45) would have appealed to Kepler who, in his *Mysterium cosmographicum* (1596) called these two principles the "duos Geometriae thesauros." See vol. 1 (1938) of his *Gesammelte Werke* (above, note 4) 41.

[53] My best thanks are due to Professor Duckworth for kindly acquainting me with the principles of his discovery and to Mr. Gabriel C. Austin of the Information Division in the New York Public Library, not only for some references, but also for calling my attention to Hermann Graf, *Bibliographie zum Problem der Proportionen, Teil* 1: *Von* 1800 *bis zur Gegenwart* (Speyer 1958), which embraces architecture, art, and nature, but not literature.